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THEORY OF SECOND HARMONIC ELECTRON CYCLOTRON RESONANCE HEATING --ETC(U)  
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NRL Memorandum Report 4101

# Theory of Second Harmonic Electron Cyclotron Resonance Heating of Tokamak Plasma

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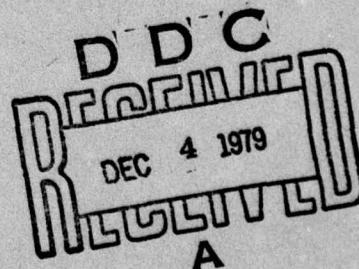
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October 26, 1979

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NRL Memorandum Report 4101	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER <b>9</b>
4. TITLE (and Subtitle) <b>6</b> THEORY OF SECOND HARMONIC ELECTRON CYCLOTRON RESONANCE HEATING OF TOKAMAK PLASMA.	5. TYPE OF REPORT & PERIOD COVERED Interim report on a continuing NRL Problem	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) <b>10</b> Bertram Hui, Edward Ott*, K. R. Chu and Thomas Antonsen**	8. CONTRACT OR GRANT NUMBER(s)	
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Research Laboratory Washington, D.C. 20375	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS NRL Problem <b>7</b> H02-37	
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Department of Energy Washington, D.C. 20545	12. REPORT DATE <b>11</b> 26 October 1979	13. NUMBER OF PAGES 15
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) <b>18</b> SBIE <b>19</b> AD-E000 336	15. SECURITY CLASS. (of this report) UNCLASSIFIED	16a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) <b>12</b> 76 Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) <b>14</b> NRL-MR-4101		
18. SUPPLEMENTARY NOTES *Also at University of Maryland, College Park, MD. **Massachusetts Institute of Technology, Cambridge, MA. 02139		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) R. F. Heating Cyclotron resonance Tokamaks		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A theoretical and numerical study of tokamak plasma heating by the second harmonic of the electron cyclotron resonance is presented. It is found that the extraordinary mode is very efficiently absorbed. In contrast to previous calculations, it is also found that the ordinary mode is well absorbed at oblique incidence to the toroidal magnetic field. This finding indicates the feasibility of heating devices of higher densities than would otherwise be possible. <b>X</b>		

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# THEORY OF SECOND HARMONIC ELECTRON CYCLOTRON RESONANCE HEATING OF TOKAMAK PLASMA

## I. INTRODUCTION

It has been shown that electron cyclotron resonance heating at the fundamental harmonic could be very promising for heating plasma to ignition for future tokamak reactors.<sup>1-4</sup> In this paper, we shall examine the characteristics of electron cyclotron resonance heating at  $\omega = 2\omega_{ce}(0)$  using a fully relativistic ray tracing code in toroidal geometry. Here  $\omega$  is the wave frequency and  $\omega_{ce}(0)$  is the electron cyclotron frequency at the center of the plasma. If the plasma is in thermal equilibrium, the results presented here could also be applied to the study of plasma emission at  $\omega = 2\omega_{ce}(0)$  by invoking Kirkoff's Law.

The motivation for considering second harmonic heating is that the accessibility conditions allow the heating of higher density tokamaks with the second harmonic. For example, for wave vector angles not too far from normal incidence to the magnetic field, the accessibility conditions at  $\omega = \omega_{ce}(0)$  are  $\omega_{ce}^2(0) \gtrsim \omega_{pe}^2(0)$  for the ordinary mode ( $\omega_{pe}$  denotes the electron plasma frequency), and  $2\omega_{ce}^2(0) \gtrsim \omega_{pe}^2(0)$  for the extraordinary mode launched from the high magnetic field side of the tokamak (waves launched from the low magnetic field side are inaccessible). On the other hand, for heating at  $\omega = 2\omega_{ce}(0)$ , the above accessibility conditions (for near normal incidence) become  $4\omega_{ce}^2(0) \gtrsim \omega_{pe}^2(0)$  for the ordinary wave and  $2\omega_{ce}^2(0) > \omega_{pe}^2(0)$  for the extraordinary wave (see Sec. II). Note that in the extraordinary wave case the largest allowable density is the same for  $\omega = \omega_{ce}(0)$  and  $\omega = 2\omega_{ce}(0)$ , but that the latter case has the advantage that waves can be launched from either the high or the low field sides of the tokamak plasma column. More significantly, for the ordinary wave at  $\omega = 2\omega_{ce}(0)$  the largest allowable density is four times that for the ordinary wave at  $\omega = \omega_{ce}(0)$  and twice that for extraordinary waves. To put this situation in perspective we can express the plasma  $\beta(0)$  in the center of the plasma in terms of  $\omega_{pe}^2(0)/\omega_{ce}^2(0)$



$$\beta(0) = (8\%) (T/10\text{KeV}) [\omega_{pe}^2(0)/\omega_{ce}^2(0)],$$

where  $T$  is the plasma temperature (assumed equal for electrons and ions). For example, at  $T = 10\text{KeV}$ , the maximum allowable  $\beta(0)$  is 8%, 16%, and 32% for the ordinary wave at  $\omega = \omega_{ce}(0)$ , the extraordinary wave, and the ordinary wave at  $\omega = 2\omega_{ce}(0)$ , respectively. [Note that the plasma  $\beta$  averaged over the tokamak cross-section would typically be  $\beta(0)/3$ ]. The possibility of heating high density (i.e. high  $\beta$ ) tokamaks evidently depends on whether the ordinary wave at  $\omega = 2\omega_{ce}(0)$  can be effectively absorbed in the plasma interior. A major result of the present study is that this is indeed found to be the case (Secs. III and IV). This result is based on the consideration of oblique angles of incidence to the magnetic field and contrasts with previous results obtained for normal incidence.<sup>2,3</sup>

## II. ACCESSIBILITY

For simplicity, in this section we consider a straight plasma cylinder with density  $N(r)$  ( $dN/dr < 0$  and  $N(a) \equiv 0$ ), and magnetic field  $\mathbf{B} = B(r, \theta) \mathbf{z}_0$ . To simulate the toroidal vacuum field dependence on the major radius we take  $B = B_0 (1 - x/R_0)^{-1}$  where  $x = -r \cos \theta$  and  $R_0$  is the major radius of the torus. Furthermore, we assume incident waves launched at  $\theta = 0$  or  $\theta = \pi$  with no  $\theta$  component of the incident wavenumber ( $k_\theta = 0$ ). Since  $\mathbf{B}$  has no  $\theta$  component,  $k_\theta$  remains zero as the wave propagates inward. (Note that these simplifications are not made in our ray tracing calculations to be presented in Sec. III.)

To find the accessibility condition at  $\omega = 2\omega_{ce}(0)$ , we proceed as in Ref. 1 and examine the cold plasma dispersion relation. We obtain three possible reflection points,

$$X_1 = 1 \quad (1)$$

$$X_2 = (1 - S^2) (1 + Y) \quad (2)$$

$$X_3 = (1 - S^2) (1 - Y) \quad (3)$$

$X_1$  is an ordinary wave reflection point,  $X_3$  is an extraordinary wave reflection point, and  $X_2$  is an ordinary [extraordinary] wave reflection point for  $S^2 > Y(Y + 2)^{-1}$  [for  $S^2 < Y(Y + 2)^{-1}$ ].

Resonance for the extraordinary mode occurs at

$$X_r \equiv 1 - Y^2 \quad (4)$$

and mode conversion between the ordinary and extraordinary modes occurs at

$$X_c \equiv 1 + Y^2 \frac{(1 - S^2)^2}{4S^2}$$

where  $X \equiv \omega_{pe}^2/\omega^2$ ,  $Y = \omega_{ce}/\omega$ ,  $S = \sin \Theta = ck_{||}/\omega$ ,  $\Theta$  is the angle of incidence (for normal incidence  $S = 0$ ) and  $k_{||}$  denote the component of  $\mathbf{k}$  parallel to  $\mathbf{B}$ .

In order for the wave to be accessible to the center of the plasma, there must be no reflection, resonance, or mode conversion points between the plasma edge and the center of the plasma (where maximum absorption occurs). By examination of the expressions for  $X_1$ ,  $X_2$ ,  $X_3$ ,  $X_r$ , and  $X_c$ , we find that this can be achieved for  $Y(0) = 1/2$  if

$$X(0) \gtrsim (1 - S^2)/2, \quad (6)$$



for the extraordinary mode, and if

$$X(0) \gtrsim \begin{cases} 1 & \text{for } S^2 \gtrsim 1/3 \\ 3/2(1 - S^2) & \text{for } S^2 \lesssim 1/3 \end{cases} \quad (7)$$

for the ordinary mode. These accessibility conditions are displayed graphically in Figs. 1 and 2.

Note that, in contrast to heating at the fundamental harmonic, the extraordinary mode is accessible from both the high field and low field sides.

For the extraordinary mode, the highest density accessible is  $\omega_{pe}^2(0) \gtrsim 2 \omega_{ce}^2(0)$ , which is no improvement over heating at  $\omega = \omega_{ce}(0)$ . For the ordinary mode, the density limit is  $\omega_{pe}^2(0) \gtrsim 4 \omega_{ce}^2(0)$ , which is probably sufficient for any future tokamak.



### III. NUMERICAL CALCULATION OF LINEAR ABSORPTION

To find out how efficient the heating at the second harmonic is, we use a relativistic ray tracing code previously developed and described in Ref. 1 to study the linear absorption of a single pass of both the ordinary and the extraordinary modes in toroidal geometry. The density and temperature profiles are assumed to be parabolic.

Two types of tokamaks are studied: (A) "reactor size", an examples is the UWMAK III (minor radius = 270 cm., major radius = 810 cm); and (B) "present-day size", an example is the Princeton Large Torus (minor radius = 40 cm., major radius = 132 cm.). The results of the calculations are summarized in Table I and II.

We see that the extraordinary mode is always efficiently absorbed for the range of angles considered, i.e., close to perpendicular incidence. This high absorption agrees with previous theoretical predictions.<sup>2,3</sup>

More importantly, our results show that the second harmonic ordinary mode is also quite efficiently absorbed (especially for "reactor-size tokamaks") at oblique incidence (an estimate of the ordinary mode absorption for slab geometry at oblique incidence is presented in the next section). Even at exactly normal incidence, the second harmonic ordinary mode absorption is larger than previous predicted<sup>2,3</sup> (probably due to a combination of ray wandering and relativistic effects which are not included in the analyses of Refs. 2,3).

## IV. CALCULATION OF THE ORDINARY WAVE TRANSMISSION COEFFICIENT

The absorption of wave energy at the second harmonic occurs mainly as a result of the coupling of the electron gyromotion with the circular component of the wave field that rotates in the electron direction. For this reason the damping of a perpendicularly propagating ordinary wave, for which the perpendicular electric field is zero, is much smaller than that of the extraordinary mode. However, if the ordinary wave is propagating at some finite angle with respect to the perpendicular, it acquires a perpendicular component of the electric field; and therefore, it becomes more highly damped.

To calculate this damping we evaluate the transmission coefficient for propagation through an  $\omega = 2\omega_{ce}$  resonant layer,

$$T = \exp(-2\pi\eta)$$

where  $\pi\eta = \int_{-\infty}^{\infty} dx k_i(x)$  is the optical depth,

$$k_i(x) = \text{Im}(\mathbf{E}_o^+ \cdot \mathbf{D}_1(k_o, x) \cdot \mathbf{E}_o) (\mathbf{E}_o^+ \cdot \frac{\partial}{\partial k_o} \mathbf{D}_o(k_o) \cdot \mathbf{E}_o)^{-1} \quad (8)$$

is the imaginary part of the local wave vector<sup>2</sup>,  $\mathbf{E}_o$  and  $k_o$  are the wave electric field and wave vector as determined from the cold plasma dielectric tensor  $\mathbf{D}_o$ , and  $\mathbf{D}_1(k_o, x)$  is the imaginary part of the correction to  $\mathbf{D}_o$  resulting from the finite Larmor radius effect at the second harmonic.

To determine  $k_i(x)$  we assume  $\mathbf{k} \cdot \mathbf{B} \ll |k||B|$  so that the perpendicular component of the electric field is small compared with the parallel component. We then find<sup>2</sup>

$$\mathbf{E}_o^+ \cdot \mathbf{D}_1 \cdot \mathbf{E}_o = \frac{1}{4} |E_-|^2 \frac{\omega_{pe}^2}{\omega_{ce}^2} \frac{k_o^2 T_e}{m_e \omega_{ce}^2} \left( \frac{X}{L} + i0 \right)^{-1}$$

and,

$$\mathbf{E}_o^+ \cdot \frac{\partial}{\partial k_o} \mathbf{D}_o \cdot \mathbf{E}_o = \frac{1}{2} |E_{||}^2| k_o c^2 / \omega_{ce}^2$$

where  $E_{||}$  is the parallel component of the wave electric field,  $E_-$  is the relevant circularly polarized component of the perpendicular electric field, and  $L$  is the scale length of the mag-



inhomogeneity,  $L^{-1} \equiv |\nabla B/B|$ . The quantity  $E_-$  can be calculated in terms of  $E_{||}$  from the cold plasma dielectric tensor<sup>6</sup>

$$|E_-/E_{||}|^2 \cong \frac{1}{2} \frac{k_{||}^2 k_o^2 c^4}{\omega^4} \left| \left( \frac{k_o^2 c^2}{\omega^2} - K_r \right) \left( K_r K_l - \frac{1}{2} \frac{k_o^2 c^2}{\omega^2} (K_r + K_l) \right) \right|^{-1/2},$$

where  $K_r = 1 - \omega_{pe}^2/(6\omega_{ce}^2)$ ,  $K_l = 1 - \omega_{pe}^2/(2\omega_{ce}^2)$ ,

and  $k_o^2 c^2/\omega^2 \cong 1 - \omega_{pe}^2/(4\omega_{ce}^2)$ . Clearly as  $k_{||} \rightarrow 0$ ,  $E_- \rightarrow 0$ . Upon performing the integration of  $k_i(x)$  over  $x$  we find,

$$\eta = \frac{\omega_{pe}^2 T_e}{2\omega_{ce}^2 m_e c^2} L k_o |E_-|^2 / |E_{||}|^2. \quad (9)$$

which applies for  $k_{||}/k_o < 1$ . In general we must add to Eq. (9) the small contribution that occurs for exactly perpendicular propagation<sup>2</sup>. However, the contribution given in Eq. (9) will be larger than this term by a factor of order  $(k_{||}/k_o)^2 [T_e/m_e c^2]^{-1}$ .

A slab model numerical code was used to test Eq. (9). The analytical and numerical results were found to be in excellent agreement ( $\leq 2\%$  discrepancy) with  $\Theta \leq 20^\circ$ , i.e. when  $k_{||} \ll k_o$ . One notes from Eq. (9), however, that  $\eta(\propto k_{||}^2)$  is a very sensitive function of the angle  $\Theta$ . In a tokamak because of the complicated geometrical factors, this angle varies considerably from point to point in the path of the wave. Consequently, while Eq. (9) serves to demonstrate the possibility of strong ordinary wave absorption at the second cyclotron harmonic, it has limited applicability to systems where  $k_{||}$  is not conserved. This indicates the necessity of utilizing the ray tracing code as we have reported in the previous section.



## V. CONCLUSION AND DISCUSSION

Generally speaking, if the plasma temperature is above 1 keV, both the ordinary and the extraordinary modes can be efficiently absorbed at either the first or the second cyclotron harmonic. For  $T_e > 1\text{keV}$ , the determining factor in regard to the choice of mode is the ratio of  $\omega_{pe}(0)$  to  $\omega_{ce}(0)$ . Table III summarizes the possible methods of heating in various regimes of  $\omega_{pe}(0)/\omega_{ce}(0)$ . In the regime  $\omega_{pe}(0) < \omega_{ce}(0)$ , ordinary mode heating at the first cyclotron harmonic frequency is the most desirable because it allows wave injection from the low field side. In the regime  $\omega_{ce}(0) < \omega_{pe}(0) < \sqrt{2}\omega_{ce}(0)$ , ordinary mode can no longer be employed and there is a trade-off between the extraordinary mode at the first harmonic and the extraordinary or ordinary mode at the second harmonic. Either choice has a disadvantage, namely, the former mode has to be launched from the high field side while the latter modes require a higher wave frequency. In the regime  $\sqrt{2}\omega_{ce}(0) < \omega_{pe}(0) < 2\omega_{ce}(0)$ , the only choice is the ordinary mode at the second harmonic. From the above consideration, it is clear that electron cyclotron resonance heating at the second harmonic frequency is a viable and possibly the only alternative for reactor size tokamaks which generally have  $\omega_{pe}(0) > \omega_{ce}(0)$ . The main difficulty involved in second cyclotron harmonic heating is the development of a high frequency wave source. In the case of UWMAK III, the required wave frequency is 225 GHz. In view of recent progress made in the gyrotron research, however, this does not appear to be an insurmountable problem. For example, a 100 GHz, 1 MW gyrotron with 30% efficiency has already been reported<sup>7</sup> and an experiment aimed at 240 GHz, 20 kW, and 25% efficiency is currently under way at the Naval Research Laboratory. Thus, there is a reasonable prospect for obtaining the required wave source in the next generation of gyrotrons.

Relatively large emission of the second harmonic ordinary and extraordinary mode have been observed.<sup>8,9</sup> The emission level of the second harmonic ordinary mode is much larger than would be predicted on the basis of previous theoretical results, which only consider normal incidence and neglect relativistic effects. On the other hand, our results are consistent with

experimental observations.

# ACKNOWLEDGMENTS

This work was supported by the Department of Energy.

## REFERENCES

1. E. Ott, B. Hui and K.R. Chu, submitted to Phys. Fluids.
2. T.M. Antonsen and W.M. Manheimer, Phys. Fluids, **21**, 2295 (1978).
3. Other works on second harmonic: W.M. Manheimer in Infrared and Millimeter Waves, Vol. 2 Edited by K. Button (Academic Press, New York) to be published; S.M. Wolfe, D.R. Cohn, R.J. Temkin and K. Kreischer, Nucl. Fusion **19**, 389 (1979).
4. Recent theoretical works on first harmonic: V. Alifanov, Yu. N. Dnestrovskii, V.V. Parail and G.V. Pereverzev, Fiz. Plazmy **3**, 230 (1977) [Sov. J. Plasma Phys. **3**, 127 (1977)]; A.G. Litvak, G.V. Permutin, E.V. Savorov, and A.A. Frajman, Nucl. Fusion **17**, 659 (1977); I. Fidone, G. Granata, G. Ramponi, and R.L. Meyer, Phys. Fluids **21**, 645 (1978); O. Eldridge, W. Namkung, and A.C. England (to be published); T. Maekawa, S. Tanaka, Y. Terumichi, and Y. Hamada, Phys. Rev. Lett. **40**, 1379 (1978).
5. R.W. Conn and G.L. Kulcinski, Electric Power Research Institute Report ER-368 (1976).
6. W.P. Allis, S.J. Buchsbaum, and A. Bers, *Waves in Anisotropic Plasmas*, MIT Press, Cambridge, Mass. (1963).
7. A.A. Andronov, V.A. Flyagin, A.V. Gaponov, A.L. Gol'denberg, M.I. Petelin, V.G. Usov, V.K. Yulpatov. "The Gyrotron — high-power source of millimetre and submillimetre waves". International Symposium on Submillimetre Waves and their Applications, Guildford, England, March 1978.
8. A. Cavallo and M. Tutter, Max-Planck Institute Report # IPP 2/244, 1978.
9. D. Boyd, private communications.



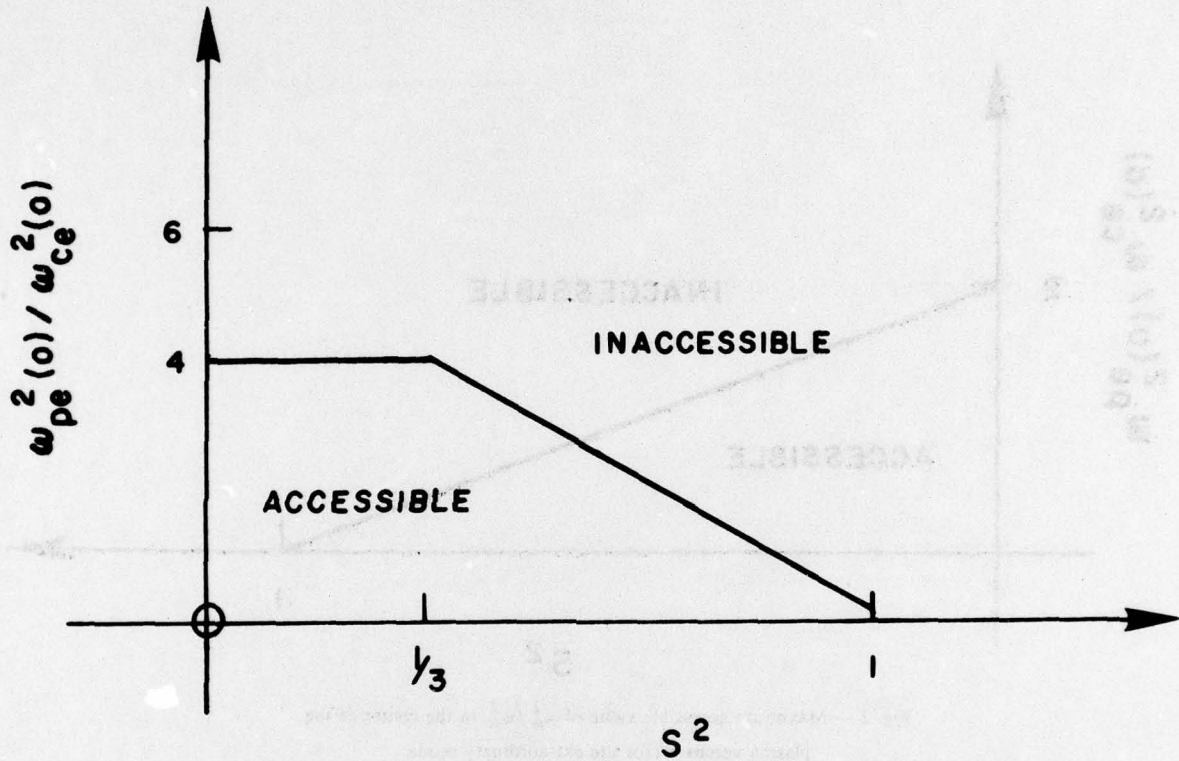


Fig. 1 — Maximum accessible value of  $\omega_{pe}^2 / \omega_{ce}^2$  in the center of the plasma versus  $S^2$  for the ordinary mode.

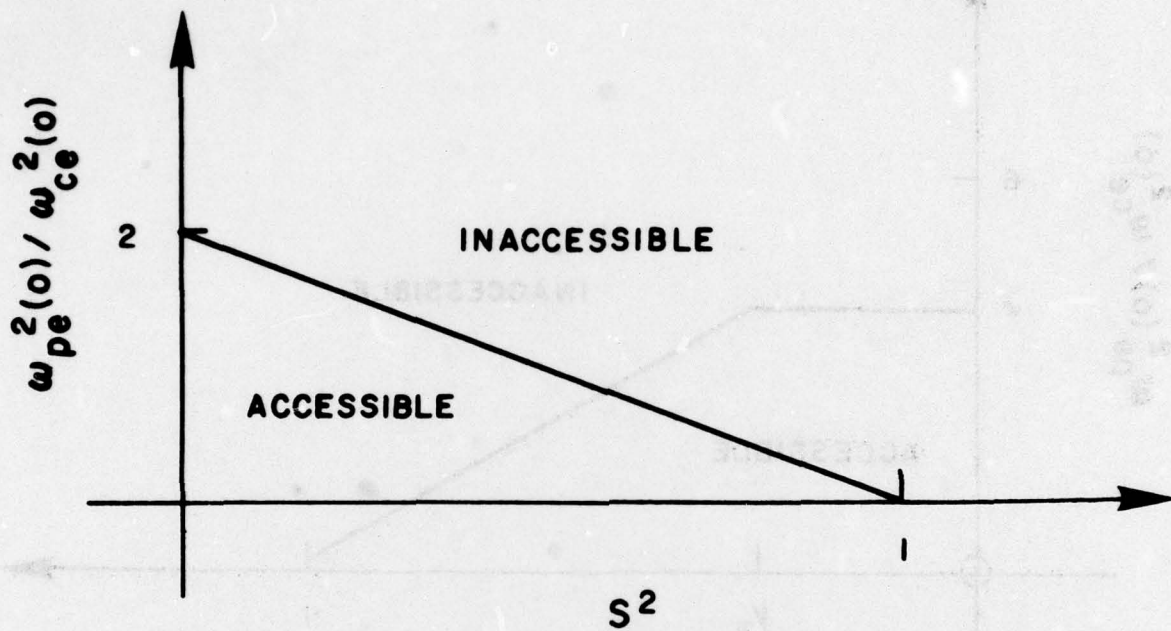


Fig. 2 — Maximum accessible value of  $\omega_{pe}^2/\omega_{ce}^2$  in the center of the plasma versus  $S^2$  for the extraordinary mode.

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Table I — Transmission Characteristics of "Reactor-size" tokamak. The magnetic field and electron temperature in the center are  $B_0 = 4.05 T$ , and  $T_0 = 2 \text{ keV}$ .  $N_0$  denotes the density in the center of the device.

Mode	$\Theta$	$\omega_{pe}(0)/\omega_{ce}(0)$	Tran. Coeff.	$N_0 (\text{cm}^{-3})$
02	0°	1.9	0.91	$5.77 \times 10^{14}$
02	10°	1.9	0.28	$5.77 \times 10^{14}$
02	20°	1.9	0.07	$5.77 \times 10^{14}$
02	0°	1.3	0.53	$2.7 \times 10^{14}$
02	10°	1.3	0.0	$2.7 \times 10^{14}$
02	20°	1.3	0.0	$2.7 \times 10^{14}$
X2	10°	1.3	0.0	$2.7 \times 10^{14}$
X2	20°	1.3	0.0	$2.7 \times 10^{14}$

Table II — Transmission Characteristics of "Present-day size" tokamak.  $\omega_{pe}(0)/\omega_{ce}(0) = 0.71$ ,  $T_0 = 2 \text{ keV}$ ,  $B_0 = 3.2 T$ , and  $N_0 = 5 \times 10^{13} \text{ cm}^{-3}$ .

Mode	$\Theta$	Tran Coeff.
02	10°	0.68
02	20°	0.60
X2	5°	0.0
X2	30°	0.0

Table III — Methods for electron cyclotron resonance heating in various plasma density regimes

Regime	Mode/Harmonic	Low Field Side Injection	High Field Side Injection	Optimum Incident Angle
$\frac{\omega_{pe}(0)}{\omega_{ce}(0)} < 1$	01	yes	yes	perpendicular
	X1	no	yes	oblique
	02	yes	yes	oblique
	X2	yes	yes	perpendicular
$1 < \frac{\omega_{pe}(0)}{\omega_{ce}(0)} < \sqrt{2}$	X1	no	yes	oblique
	02	yes	yes	oblique
	X2	yes	yes	perpendicular
$\sqrt{2} < \frac{\omega_{pe}(0)}{\omega_{ce}(0)} < 2$	02	yes	yes	oblique